FINANCIAL PERFORMANCE VARIANCE ANALYSIS OF NON-LINEAR DECOMPOSITION IN METALLURGY

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Abstract

Financial performance is a random process which could be decomposed in the particular indices. Often the influence function is of a non-linear form. For a performance analysis is important knowledge of the indices influence. One of the possible random influence measure is variance. However, the problem is more complicated under non-linear function of indices. There is in the paper derived a problem of the non-linear delta method approximation of variance decomposition. Financial performance variance decomposition in metallurgy sector is verified and investigated.

Keywords: Financial performance, delta approximation method, variance decomposition

1. INTRODUCTION

Influence analysis and deviation analysis is one of the important instrument in the financial decision-making. We can distinguish the static (one-period) and dynamic (multi-period) analysis. The second aspect is characterised by the type of investigated function: linear (additive) and non-linear type. Financial performance is crucial synthetic indicator of financial and economical level. We can say the importance of the measure on all levels of economic subjects, companies, sectors, economy including companies and sector of metallurgy.

The static approaches were presented e.g. Dluhosova (2014), Dluhosova (2015), Zmeskal (2013). The objective of the paper is to derive the method for multi-period (dynamic) influence analysis for financial performance of non-linear dependency function of financial performance applicable in metallurgy sector. The method proposal is based on delta (linear) approximation by the Taylor expansion and variance analysis. Example of decomposition of financial performance on Return on Assets (ROA) measure in metallurgy is presented.

2. DERIVATION OF THE DELTA VARIANCE ANALYSIS METHOD

There is in the chapter derived the variance of non-linear function on the delta approximation. The first the linear example is derived and subsequently, the non-linear delta approximation formula is derived. The chapter includes the special example of three influential factors.

2.1 Linear variance analysis method derivation

We suppose the linear function,

$$Y = \sum_i a_i F_i ,$$

where Y is dependent variable, a_i is coefficient and F_i is independent (explanation) factor.
We can express the margin as follows,

$$\Delta Y = \sum a_i \cdot \Delta F_i , \quad a_i = E(F_i),$$

where $\Delta F_j = F_j - E(F_j)$, so

$$\Delta Y = \sum a_i \cdot \Delta F_i - E\left(\sum a_i \cdot \Delta F_i \right) = \sum a_i \cdot \Delta F_i - \sum a_i \cdot E(\Delta F_i).$$

Variance of the function is following

$$\mathrm{var}(\Delta Y) \equiv \mathrm{var}(\Delta f(F_1, F_2, \cdots, F_n)) = E(\Delta f)^2 =$$

$$= E\left(\sum_i a_i \cdot \Delta F_i \right)^2 = \sum_i a_i^2 \cdot E(\Delta F_i)^2 + \sum_i \sum_{j \neq i} a_i \cdot a_j \cdot E(\Delta F_i \cdot \Delta F_j)$$

here $E(\Delta F_i)^2 = \mathrm{var}(F_i)$, $E(\Delta F_i \cdot \Delta F_j) = \mathrm{cov}(F_i, F_j)$, it implies that

$$\mathrm{var}(\Delta Y) \equiv \mathrm{var}(\Delta f(F_1, F_2, \cdots, F_n)) = \sum_i a_i^2 \cdot \mathrm{var}(F_i) + \sum_{\substack{i \\ i \neq j}} \sum_j a_i \cdot a_j \cdot \mathrm{cov}(F_i, F_j).$$

So formulae the variance of particular factor is as follows,

$z_i = a_i^2 \cdot \mathrm{var}(F_i) + \sum_{\substack{j \\ i \neq j}} a_i \cdot a_j \cdot \mathrm{cov}(F_i, F_j)$, it stems that relative influence of particular factor is

$$s_i = \frac{z_i}{\sum_i z_i} .$$

Remark to the variance formulation: $\mathrm{var}(x) = \mathrm{var}(\Delta x)$, because $\mathrm{var}(x) = E(x - E(x))^2 = E(x^2) - E(x)^2$,
and $\mathrm{var}(\Delta x) = E(\Delta x - E(\Delta x))^2 = E(\Delta x^2) - E(\Delta x)^2 = E\left[(x - E(x))^2\right] - E[x - E(x)]^2 =$
$= E\left[x^2 - 2x \cdot E(x) + E(x)E(x)\right] = E(x^2) - E(x)^2$.

2.2 Linear approximation delta variance analysis method derivation

The basic non-linear function is
$Y = f(F_1, F_2, \cdots, F_n)$, and margin
$\Delta Y = \Delta f(F_1, F_2, \cdots, F_n)$.
Because of application of the Taylor expansion, the general formulae is

$\Delta f(F_1, F_2, \cdots, F_n) = \sum_j \frac{\partial f(\cdot)}{\partial F_j} \cdot \Delta F_j + \frac{1}{2} \sum_j \sum_k \frac{\partial^2 f(\cdot)}{\partial F_j \cdot \partial F_k} \cdot \Delta F_j \cdot \Delta F_k + \cdots$, linear part is

$$\Delta f(F_1, F_2, \cdots, F_n) = \sum_i \frac{\partial f(\cdot)}{\partial F_i} \cdot \Delta F_i .$$

The margin (residual deviation) is
$\Delta f(F_1, F_2, \cdots, F_n) = f(F_1, F_2, \cdots, F_n) - E(f(F_1, F_2, \cdots, F_n))$, then the variance formula is following,

$$\mathrm{var}(\Delta f(F_1, F_2, \cdots, F_n)) = E(\Delta f)^2 = \left(\sum_i E\left[\frac{\partial f(\cdot)}{\partial F_i} \right] \cdot \Delta F_i \right)^2 ,$$

where $\Delta F_j = F_j - E(F_j)$, $a_i = E\left[\frac{\partial f(\cdot)}{\partial F_i}\right]$.

General formula for variance

$$\mathrm{var}(\Delta Y) \equiv \mathrm{var}(\Delta f(F_1, F_2, \cdots, F_n)) =$$

$$= E(\Delta f)^2 = E\left(\sum_i a_i \cdot \Delta F_i\right)^2 = \sum_i a_i^2 \cdot E(\Delta F_i)^2 + \sum_{\substack{i \\ i \neq j}} \sum_j a_i \cdot a_j \cdot E(\Delta F_i \cdot \Delta F_j)$$

where $E(\Delta F_i)^2 = \mathrm{var}(F_i)$, $E(\Delta F_i \cdot \Delta F_j) = \mathrm{cov}(F_i, F_j)$, $a_i = E\left[\frac{\partial f(\cdot)}{\partial F_i}\right]$ and so

$$\mathrm{var}(\Delta Y) \equiv \mathrm{var}(\Delta f(F_1, F_2, \cdots, F_n)) = \sum_i a_i^2 \cdot \mathrm{var}(F_i) + \sum_i \sum_{j \neq i} a_i \cdot a_j \cdot \mathrm{cov}(F_i, F_j).$$

Therefore we can express variance of particular factor as follows,

$$z_i = a_i^2 \cdot \mathrm{var}(F_i) + \sum_{j \neq i} a_i \cdot a_j \cdot \mathrm{cov}(F_i, F_j),$$

and relative influence of particular factor is

$$s_i = \frac{z_i}{\sum_i z_i}.$$

2.2.1 Linear approximation delta variance analysis method derivation of three factors

Applying procedure of previous part, for three factors

$$\Delta Y \equiv \Delta f(F_1, F_2, F_3) = E\left[\frac{\partial f(\cdot)}{\partial F_1}\right]\Delta F_1 + E\left[\frac{\partial f(\cdot)}{\partial F_2}\right]\Delta F_2 + E\left[\frac{\partial f(\cdot)}{\partial F_3}\right]\Delta F_3,$$

where $a_i = E\left[\frac{\partial f(\cdot)}{\partial F_i}\right]$,

$$\Delta Y \equiv \Delta f(F_1, F_2, F_3) = a_1 \cdot \Delta F_1 + a_2 \cdot \Delta F_2 + a_3 \cdot \Delta F_3.$$

Variance of the function is

$$\mathrm{var}(\Delta f(F_1, F_2, F_3)) = E(\Delta f)^2 = E(a_1 \cdot \Delta F_1 + a_2 \cdot \Delta F_2 + a_3 \cdot \Delta F_3)^2 =$$

$$= \sum_i^3 a_i^2 \cdot E(\Delta F_i)^2 + \sum_i^3 \sum_{j \neq i}^3 a_i \cdot a_j \cdot E(\Delta F_i \cdot \Delta F_j) = \sum_i^3 a_i^2 \cdot \mathrm{var}(F_i) + \sum_{\substack{i \\ i \neq j}}^3 \sum_j^3 a_i \cdot a_j \cdot \mathrm{cov}(F_i, F_j) =$$

$$= \sum_i^3 E\left[\frac{\partial f(\cdot)}{\partial F_i}\right]^2 \cdot \mathrm{var}(F_i) + \sum_i^3 \sum_{j \neq i}^3 E\left[\frac{\partial f(\cdot)}{\partial F_i}\right] \cdot E\left[\frac{\partial f(\cdot)}{\partial F_j}\right] \cdot \mathrm{cov}(F_i, F_j)$$

Variance of the particular factor is

$z_i = a_i^2 \cdot \mathrm{var}(F_i) + \sum_i^3 \sum_{j \neq i}^3 a_i \cdot a_j \cdot \mathrm{cov}(F_i, F_j)$, and relative variance

$$s_i = \frac{z_i}{\sum_i z_i}.$$

3. APPLICATION OF DELTA NON-LINEAR VARIANCE METHOD IN METALLURGY SECTOR

The goal of the application is to find the relative influence of the three factors (sales profitability, assets turnover and financial leverage) on return on equity in metallurgy sector. The input are quarterly data of metallurgy sector in period 1Q 2007 to 2Q 2014.

3.1 Linear approximation delta variance analysis method formulas

We suppose the influential function of performance in the non-linear multiplicative form,

$$ROE = \frac{EAT}{S}\frac{S}{A}\frac{A}{E} = F_1 \cdot F_2 \cdot F_3,$$

where $\frac{EAT}{S}$ is sales profitability, $\frac{S}{A}$ is assets turnover, $\frac{A}{E}$ is financial leverage, E is equity, S is sales, A is asset.

So, the formulas of variance influence calculation in coincidence with previous part are

$$z_i = a_i^2 \cdot \mathrm{var}(F_i) + \sum_j a_i \cdot a_j \cdot \mathrm{cov}(F_i, F_j),$$

$$z_1 = a_1^2 \cdot \mathrm{var}(F_1) + a_1 \cdot a_2 \cdot \mathrm{cov}(F_1, F_2) + a_1 \cdot a_3 \cdot \mathrm{cov}(F_1, F_3),$$

$$z_2 = a_2^2 \cdot \mathrm{var}(F_2) + a_2 \cdot a_1 \cdot \mathrm{cov}(F_2, F_1) + a_2 \cdot a_3 \cdot \mathrm{cov}(F_2, F_3),$$

$$z_3 = a_3^2 \cdot \mathrm{var}(F_3) + a_3 \cdot a_1 \cdot \mathrm{cov}(F_3, F_1) + a_3 \cdot a_2 \cdot \mathrm{cov}(F_3, F_2),$$

where $a_1 = E\left[\frac{\partial(F_1 \cdot F_2 \cdot F_3)}{\partial F_1}\right] = E(F_2) \cdot E(F_3)$, $a_2 = E\left[\frac{\partial(F_1 \cdot F_2 \cdot F_3)}{\partial F_2}\right] = E(F_1) \cdot E(F_3)$,

$$a_3 = E\left[\frac{\partial(F_1 \cdot F_2 \cdot F_3)}{\partial F_3}\right] = E(F_1) \cdot E(F_2).$$

So formulas of particular factors are,

$$z_1 = (E(F_2) \cdot E(F_3))^2 \cdot \mathrm{var}(F_1) +$$
$$E(F_2) \cdot E(F_3) \cdot E(F_1) \cdot E(F_3) \cdot \mathrm{cov}(F_1, F_2) + E(F_2) \cdot E(F_3) \cdot E(F_1) \cdot E(F_2) \cdot \mathrm{cov}(F_1, F_3)$$
$$z_2 = (E(F_1) \cdot E(F_3))^2 \cdot \mathrm{var}(F_2) +$$
$$E(F_1) \cdot E(F_3) \cdot E(F_2) \cdot E(F_3) \cdot \mathrm{cov}(F_2, F_1) + E(F_1) \cdot E(F_3) \cdot E(F_1) \cdot E(F_2) \cdot \mathrm{cov}(F_2, F_3)$$
$$z_3 = (E(F_1) \cdot E(F_2))^2 \cdot \mathrm{var}(F_3) +$$
$$E(F_1) \cdot E(F_2) \cdot E(F_2) \cdot E(F_3) \cdot \mathrm{cov}(F_3, F_1) + E(F_1) \cdot E(F_2) \cdot E(F_1) \cdot E(F_3) \cdot \mathrm{cov}(F_3, F_2)$$

3.2 Input data

There are in the Fig. 1 presented the input quarterly data data of indices, return on equity, sales profitability, assets turnover and financial leverage.

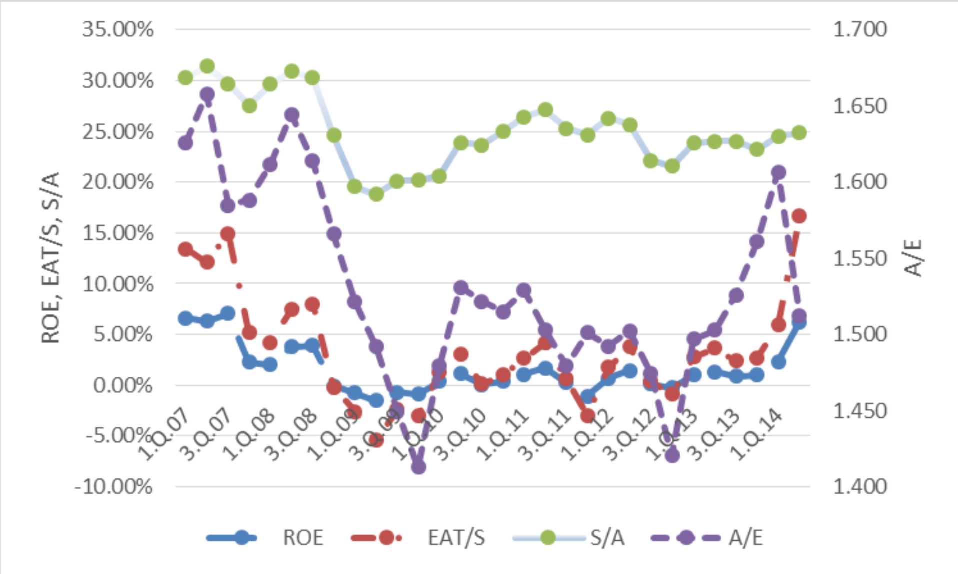

Fig. 1 input data development

4. RESULTS

There are in Tab. 1, Tab. 2 and Tab. 3 presented results of the the calculation sue to part 3.1. It is apparent that if we measure the financial performance by the ROE indicator, then the most influential is profitability sales (92.07%), less important assets turnover (6.31%) and very small influence has financial leverage (1.62%).

Tab. 1 Calculated parameters

i	1	2	3
E(Fi)	0.0335	0.2500	1.5308
a_i	0.382690735	0.05128143	0.008375018

Tab. 2 Covariance matrix

	EAT/S	S/A	A/E
EAT/S	0.0028	0.0013	0.0021
S/A	0.0013	0.0012	0.0017
A/E	0.0021	0.0017	0.0038

Tab. 3 Influence of factors

	EAT/T	T/A	A/VK	ROE
z_i	0.000441826	3.0289E-05	7.76891E-06	0.00047988
s_i	92.07%	6.31%	1.62%	100.00%

CONCLUSION

The paper was devoted to the deviation influential analysis of metallurgy sector. The delta approximation variance method of non-linear function was derived. The financial performance of the ROE indictor was verified. The most influential factor of metallurgy sector of quarterly data of period 1Q 2007 to 2Q 2014 was founded the sales profitability. The deviation analysis is important method and results of analysis can serve as the important information in managing industry sector.

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