



# MODELLING OF LONGITUDINAL ROLLING PROCEDURE OF ALUMINUM SHEET UNDER SUPERPLASTICITY CONDITIONS

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# Abstract

The two-dimensional task of isothermal rolling of aluminum sheet in the superplasticity conditions realized with a low angle of nip is considered. The solution of the task is based on the well-known mechanisms of the plastic flow in the wedge-shaped convergent canal. For the mathematical formulation of the task the theory of elasto-plastic processes ratios of the small curvature was attracted, and the state equation is suitable for the description of regularities of high-temperature deformation in the wide strain rate-rate interval including the conditions of superplasticity effect realization. The components of the movements rates vector, tensors of the stresses and strain rates are written down. The parameters summarized rate, geometrical contact factors in zones of backward and forward of rolling are determined. It is shown the existence of the angle of nip below which the center of deformation is considered only as forward of rolling zone. The specific calculations of the roll force distribution corresponds to thermal superplasticity conditions and outside of its applied to industrial alloy EN AW 5083 was carried out and the results were compared. The experimental realization of the process has been realized in the approximate chemical composition alloy EN AW Alustar with the producing fine-grained structure in the rolled product.

Keywords: superplasticity, isothermal rolling, industrial alloy EN AW 5083, industrial alloy EN AW Alustar

# 1. INTRODUCTION

Metal forming processes of bulk materials are based, as a rule, on the force impact on the deformable material. At the shaping under thermomechanical conditions of superplasticity the unique properties of metals and alloys consisting in sharp decrease of resistance to deformation can be used. Superplasticity at the same time is considered [1] as a special state of the polycrystalline material plastically deformed at the low level of the stress with the retaining of the ultrafine-grained structure – structural superplasticity produced at the previous stage or arised during hot deformation independently from the initial grain size – dynamic superplasticity [1,2] . For both types of superplasticity are supposed the domination of grain-boundary sliding over the other mass-transfer mechanisms [3]. Consequently, for realization of the dynamic superplasticity it has to substitute an initial structural condition of material by another one, allowed to realize a superplasticity. For industrial aluminum alloys the mentioned above substitution takes place in temperature-strain-rate conditions of dynamic recrystallization [4,5].

# 2. STATEMENT OF THE PROBLEM

Let's consider a two-dimensional task of hot rolling of a sheet in the rolls of identical radius of R (Fig. 1) rotating with identical angular velocities. It is supposed that process of rolling is carried out with a low angle of nip. It means that for establishment of power and kinematic parameters of operation research [6-8] of a current



of material in the wedge-shaped meeting canal with a corner at top of  $\alpha_1$  (Fig. 1) can be used. It is supposed that process of rolling is realized in isothermal conditions.

Cylindrical system of coordinates of  $\rho \alpha z$  it will enter and consider the beginning of coordinates in wedge top. All geometrical sizes we believe carried to sheet width.



Fig. 1 Rolling process schematization

The mathematical problem definition within the theory of elasto-plastic processes ratios of the small curvature is given in [1].

## 3. THE DECISION FUNCTION

Integrating an incompressibility condition in rates, we receive:

$$\nu_{\rho} = \frac{k(\alpha)}{\rho} , \qquad (1)$$

where  $v_{\rho}$  – a radial projection of a rate vector of movements;  $k = k(\alpha)$  – the unknown function which is subject to definition.

Having used the decision (1), for the making speeds of deformations we can write down:

$$\dot{\varepsilon}_{\rho} = -\frac{k(\alpha)}{\rho^2}; \ \dot{\varepsilon}_{\alpha} = \frac{k(\alpha)}{\rho^2}; \ \dot{\gamma}_{\rho\alpha} = \frac{k'(\alpha)}{\rho^2}.$$
(2)

Intensity of deformations rates in the conditions of a radial current can be defined in a look:

$$\dot{c}_{\mu} = \frac{1}{\rho^2} e^{-2}(\alpha), \ L(\alpha) = \frac{1}{3} \Big[ 4k^2(\alpha) + k'^2(\alpha) \Big].$$
(3)

The defining ratios [1, 9] with use the equations (2), (3) we will write down as follows:

$$\sigma_{\rho} - \sigma_{0} = -\frac{2}{3}T(\alpha, \rho)k(\alpha); \ \sigma_{\alpha} - \sigma_{0} = \frac{2}{3}T(\alpha, \rho)k(\alpha); \ \tau_{\rho\alpha} = \frac{1}{3}T(\alpha, \rho)k'(\alpha), \tag{4}$$



where 
$$T(\rho, \alpha) = (1 - m_0 - \beta)L^{1/2}(\alpha) + \frac{3m_0 + \beta}{\rho^2} - \frac{3m_0}{\rho^4}L^{1/2}(\alpha) + \frac{m_0}{\rho^6}L(\alpha)$$
.

The analysis of equations (1) – (4) shows that components of tension, speeds of movements and deformations will be found if the obvious type of function  $k = k(\alpha)$  is established which, as well as in [9], we will call defining. For search of function  $k(\alpha)$  we will substitute dependences (4) in the equations of balance [1]. Also we will differentiate the received derivatives  $\partial \sigma_0 / \partial \rho$  and  $\partial \sigma_0 / \partial \alpha$  respectively on  $\alpha$  and  $\rho$ . We will equate the right parts of the received mixed derivatives each other. As a result we will receive the differential equation

$$k''' + 4k' = 0. (5)$$

With due regard for two obvious boundary conditions  $\tau_{\rho\alpha}|_{\alpha=0} = 0$  and  $\tau_{\rho\alpha}|_{\alpha=\frac{\alpha_1}{2}} = -\chi \tau_{max}|_{\alpha=\frac{\alpha_1}{2}}$  the solution of the equation (5) can be submitted in form

$$k(\alpha) = \frac{c}{2}(\psi - \cos 2\alpha), \tag{6}$$

where  $\chi$  – the experimental coefficient [7] establishing conditions of contact of rolls and a deformable sheet; *c* – is a constant, and for the  $\psi(\alpha_1, \chi)$  is received:

$$\psi(\alpha_1, \chi) = \frac{\sqrt{1-\chi^2}}{\chi} \sin \alpha_1 - \cos \alpha_1.$$
(7)

Definition of the constant *c* it is compatible to a choice of the center of plastic deformation which, following [7,9], takes the wedge-shaped form limited to two surfaces of a rupture of speeds of  $\rho_1 = \rho_1(\alpha_1)$ ;  $\rho_2 = \rho_2(\alpha_1)$  respectively on an entrance to rolls and at the exit (Fig. 1) from them. Procedure of the analysis of the center of deformation is in detail lit in [9] and allows to establish the allowing function  $k = k(\alpha)$  and the  $\rho_1(\alpha)$ ,  $\rho_2(\alpha)$  function limiting the center of plastic deformation in the radial direction:

$$k(\alpha) = \frac{\upsilon_1 h_1}{\overline{\psi}} (\psi - \cos 2\alpha), \ \rho_1(\alpha) = \frac{h_1}{2\overline{\psi}} \cdot \frac{2\psi\alpha - \sin 2\alpha}{\sin \alpha}; \ \rho_2(\alpha) = (1 - \Lambda) \frac{h_1}{2\overline{\psi}} \cdot \frac{2\psi\alpha - \sin 2\alpha}{\sin \alpha}.$$
(8)

Here  $\Lambda = h_2/h_1$  – sinking of a sheet (Fig. 1);  $v_1$  – the average movement rate of material on an entrance to rolls;

$$\overline{\psi}(\alpha_1, \chi) = \frac{\alpha_1}{\psi \alpha_1 - \sin \alpha_1}.$$
(9)

#### 4. FORCE PARAMETERS OF ROLLING

Components of a tensor of tension are defined on the basis of the solution of the differential equilibrium equations [1,10] together with (2), (4), (6):



$$3\sigma_{\rho} = (1 - m_{0} - \beta)L^{-1/2} \left(\frac{k'L'}{2L} - k'' + 4k\right) \ln \frac{\rho}{\rho_{2}} - 4(1 - m_{0} - \beta)L^{-1/2}k - \frac{3m_{0} + \beta}{2} (k'' - 4k) \left(\frac{1}{\rho_{2}^{2}} - \frac{1}{\rho^{2}}\right) - 4(3m_{0} + \beta)\frac{k}{\rho_{2}^{2}} + \frac{3}{4}m_{0}L^{1/2} \left(\frac{k'L'}{2L} + k'' - 4k\right) \left(\frac{1}{\rho_{2}^{4}} - \frac{1}{\rho^{4}}\right) + 12m_{0}L^{1/2}\frac{k}{\rho_{2}^{4}} - \frac{m_{0}}{6}L \left(\frac{k'L'}{L} + k'' - 4k\right) \left(\frac{1}{\rho_{2}^{6}} - \frac{1}{\rho^{6}}\right) - 4m_{0}L\frac{k}{\rho_{2}^{6}};$$

$$3\sigma_{\alpha} = (1 - m_{0} - \beta)L^{-1/2} \left(\frac{k'L'}{L} - k'' + 4k\right) \ln \frac{\rho}{\rho_{2}} - \frac{3m_{0} + \beta}{2} (k'' + 4k) \left(\frac{1}{\rho_{2}^{2}} - \frac{1}{\rho^{2}}\right) + \frac{3}{4}m_{0}L^{1/2} \left(\frac{k'L'}{2L} + k'' + 12k\right) \times \left(\frac{1}{\rho_{2}^{4}} - \frac{1}{\rho^{4}}\right) - \frac{m_{0}}{6}L \left(\frac{k'L'}{L} + k'' + 8k\right) \left(\frac{1}{\rho_{2}^{6}} - \frac{1}{\rho^{6}}\right); \quad 3\tau_{\rho\alpha} = \left[(1 - m_{0} - \beta)L^{-1/2} + \frac{3m_{0} + \beta}{\rho^{2}} - \frac{3m_{0}}{\rho^{4}}L^{1/2} + \frac{m_{0}}{\rho^{6}}L\right]k. \quad (10)$$

It is natural to consider that the longitudinal forces arising on an entrance to the deformation center ( $N_1$ ) and their exit it ( $N_2$ ), be reduced to zero. Therefore, we can write down

$$N_{1} = 2 \int_{0}^{\alpha_{1/2}} \sigma_{\rho} \Big|_{\rho = \rho_{1}} dA = 0; \qquad N_{2} = 2 \int_{0}^{\alpha_{1/2}} \sigma_{\rho} \Big|_{\rho = \rho_{2}} dA = 0.$$
(11)

The second condition (11) after substitution of expression for  $\sigma_{\rho}$  from (10) will be transformed to it to the cubic equation of a look

$$a_1 + a_1 \mu_{0n} + a_2 \mu_{0n}^2 + a_3 \mu_{0n}^3 = 0,$$
(12)

where coefficients  $a_i$  are functions of the angle of nip;  $\mu = \mu_{0n}$  it is necessary to consider as the parameter generalizing high-rate, geometrical and contact factors at the exit from a zone of an advancing and determined by expression

$$\mu_{0n} = \frac{\nu_{\rm l} \overline{\psi}}{h_{\rm l} \left(1 - \Lambda\right)^2} \,. \tag{13}$$

The first condition (11) on an entrance to the deformation center (in a lag zone where  $\mu = \mu_{0T}$ ) it is led to the equation

$$b_0 + b_1 \mu_{0T} + b_2 \mu_{0T}^2 + b_3 \mu_{0T}^3 = 0.$$
(14)

For coefficients  $b_i$  analytical expressions which interpretation isn't given are received.

The size of sinking of a deformable sheet is set for technological reasons and is connected with the angle of nip  $\alpha_1$  and diameter of rolls 2R [8,11]. Expression is a consequence of the law of change of thickness of a sheet in the center of deformation

$$\cos \alpha_1 = 1 - \frac{h_1}{2R} \Lambda \,. \tag{15}$$



The numerical solution of the equations (12) and (14) with attraction (15) is received at R = 0.11;  $\chi = 0.3$  and three values of thickness  $h_1$  equal 0.015; 0.03 and 0.07, and also at  $m_0 = 0.339$ ,  $\beta = -0.134$ , corresponding an alloy EN AW 5083. Results of calculations are given in Fig. 2 in which curves 1, 2, 3 are functions  $\mu_{0T}(\alpha_1)$  respectively for  $h_1 = 0.015$ ; 0.03 and 0.07, and a curve  $4 - \mu_{0n}(\alpha_1)$ .



**Fig. 2** Dependence of  $\mu$  parameter of the angle of nip  $\alpha_1$ 

From the presented diagramss it is visible that with growth of thickness of initial sheet there is a alignment of dependences  $\mu_{0T}(\alpha_1)$  to approach to a curve 4.

We identify pressure upon rolls  $q = q(\rho)$  with district normal stress on a contact surface

$$\sigma_{\alpha}\big|_{\alpha=\alpha_1/2} = -q \ . \tag{16}$$

Substitution in (16) expressions for  $\sigma_{\alpha}$  from (10) for the value of pressure upon rolls we receive

$$q = -\frac{1}{3} \left\{ n_0 \ln \frac{\rho}{\rho_2} + n_1 \left( \frac{1}{\rho_2^2} - \frac{1}{\rho^2} \right) + n_2 \left( \frac{1}{\rho_2^4} - \frac{1}{\rho^4} \right) + n_3 \left( \frac{1}{\rho_2^6} - \frac{1}{\rho^6} \right) \right\},$$
(17)

where coefficients  $n_i$  are functions of the angle of nip and rate of supply of material in rolls.

In Fig. 3 *a,b* the dependences of pressure upon q rolls on the radius of  $\rho \in (\rho_2; \rho_1)$  received as a result of calculations for formula (17) are given as an example, and to the radius of  $\rho_2(\alpha_1)$  there correspond zero on abscissa axis, and sizes specified in the field of drawing are sinkings of a strip of  $\Lambda$ . Calculations are perfromed at an initial thickness of sheet 0.015. Qualitatively epyura of pressure don't contradict known data [9] and are constructed in temperature conditions of superplasticity ( $\beta = -0.134$ ) and out of them ( $\beta = 0.236$ ).





Fig. 3 Dependence of pressure upon rolls on radius  $\rho$  for different sinkings of a sheet 0.015 thick for: a)  $\beta$  =-0.134; b)  $\beta$  =0.236

## CONCLUSION

- The problem of the force parameters determination when rolling a sheet of aluminum alloy EN AW 5083 at the superplasticity thermal conditions has been solved.
- The definition and optimisation of the superplasticity area in the center of deformation location was realized.

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